20 Years of Safe Withdrawal Rate Research

Executive Summary

- The safe withdrawal rate research began nearly 20 years ago, when Bill Bengen published an article in 1994 examining how spending would have been sustained looking not at average rates of return, but actual historical sequences of market returns. He showed a withdrawal rate of 4% of the initial account balance, with the withdrawal dollar amounts adjusted for inflation in each future year, was capable of surviving any 30-year sequence found in history, using a 50/50 asset allocation. This withdrawal rate is reduced by 0.2% if the client wishes to leave the original principal as a legacy for clients.

- In the two decades since Bengen's original research, numerous studies have expanded the safe withdrawal rate framework to incorporate additional factors.

- The original 4% safe withdrawal rate was based on a 30-year time horizon. Increase the safe withdrawal rate by 1% for a 20-year time horizon (and make the portfolio more conservative); decrease the safe withdrawal rate by 0.5% for a 40-year time horizon.

- Greater diversification increases the safe withdrawal rate, although the exact amount of the increase is difficult to determine due to limited return history for some asset classes. An increase of 0.5% to 1.0% appears prudent for well diversified portfolios.

- Asset allocation glide paths in retirement - where the portfolio steadily gets more conservative over time - may not necessarily be helpful. The net effect is to reduce average equity exposure - and therefore, growth - over time, and can actually have a detrimental effect.

About the Author

Michael E. Kitces, MSFS, MTAX, CFP®, CLU, ChFC, RHU, REBC, CASL, is the Director of Research for Pinnacle Advisory Group (www.pinnacleadvisory.com), a private wealth management firm located in Columbia, Maryland. In addition, he is an active writer and speaker, and publishes The Kitces Report and his blog "Nerd's Eye View" through his website www.kitces.com.

- Expenses reduce the safe withdrawal rate, but not to the extent many anticipate. Since expenses decline as a portfolio declines, and taxes themselves take a bite out of growth (leaving less to pay expenses on), the safe withdrawal rate is reduced by only about 35% of the associated expense ratio (e.g., the safe withdrawal rate goes down 0.35% for every 1% of expenses). The safe withdrawal rate may also rise in a similar fashion for the "superinvestor" who generates alpha in the portfolio.

- Taxes reduce the safe withdrawal rate, as cumulative growth is dragged down over time. Reduce the safe withdrawal rate by 0.25% for low-taxation clients, 0.5% for moderate-taxation clients, and 0.75% for high-taxation clients. In situations where the withdrawals occur from a pre-tax account, simply reduce the entire gross withdrawal amount by the associated tax rate.

- Clients with spending flexibility can sustain higher initial withdrawal rates, assuming they are in fact capable of adjusting spending in the face of a difficult market or economic environment. Increase the safe withdrawal rate by 0.5% to 1.0% for clients willing to adjust spending; further increases are merited for clients who have a high risk tolerance.

- Market valuation at the time of retirement can impact market returns over an extended period of time, which in turn impacts safe withdrawal rates. When market valuation is at average or favorable levels, increase the safe withdrawal rate by 0.5% to 1.0%; at unfavorable valuation, use the existing baseline. A willingness to tactically adjust the portfolio in light of valuation changes can add an additional 0.2% to the withdrawal rate.

- The research in support of annuities in conjunction with safe withdrawal rates is mixed. Research suggests that there may be some value to a partial immediate annuitization, but the case is less clear for variable annuities with guaranteed income riders.

Ultimately, the most effective safe withdrawal rate solution should use the various research factors to adjust the client's own safe withdrawal rate upwards or downwards, based on how the factors apply to the client's own circumstances.

Introduction

It's been almost 20 years since Bill Bengen first published "Determining Withdrawal Rates Using Historical Data" in the October 1994 issue of the Journal of Financial Planning, spurring a new body of research focused on determining what amount is safe and sustainable to withdraw from a portfolio over an extended period of time.

In the two decades that have followed since Bengen's seminal article, this line of retirement income research has added many new layers, introducing both refinements and complexity to the original safe withdrawal rate framework. The end result gives a more complete picture of what is and is not sustainable spending, but in the process has also led to significant confusion, misapplication, and a great deal of criticism.

In this month's newsletter, we look back and review the past 20 years of safe withdrawal rate literature, in an effort to better understand the major revelations and research innovations, and arrive at some conclusions about what this entire body of research can tell us today about what is and isn't a safe, sustainable portfolio withdrawal.

It All Started With Bengen

The safe withdrawal rate research started with Bill Bengen's "Determining Withdrawal Rates Using Historical Data" in the October 1994 issue of the Journal of Financial Planning. However, as Bengen wrote, he actually viewed his research as an extension of an article from the Journal of Financial Planning earlier that year, entitled "Investing for Retirement: Using the Past to Model the Future" by Larry Bierwirth. The basic principle was relatively straightforward: instead of using historical *averages* to evaluate a client's retirement plan, projections should instead be based on *actual* results that had occurred at some prior point in history. In other words, a client's plan could be tested by looking not just at what happens on average, but what happens if the client retired in a particular year, such as 1926 or 1967.

While Bierwirth's intention was to use the historical scenarios approach to evaluate a client's particular plan (as opposed to just using long-term historical averages), Bengen had a different goal: to determine what kind of "portfolio longevity" was associated with various withdrawal amounts relative to the starting portfolio. The ultimate goal was to figure out what amounts can be safely withdrawn for a particular time horizon under any historical scenario, so planners can give clients prospective spending advice based on their own individual (retirement) time horizons.

Accordingly, Bengen drew on the available returns published in the Ibbotson Associates' *Stocks, Bonds, Bills and Inflation: 1992 Yearbook* to determine safe spending. Initially, he evaluated what spending level would be safe over 50 years, finding that an initial withdrawal rate as high as 3.5% (subsequently adjusted each year for inflation) allowed a 50/50 portfolio to go the entire time period without running out of money. If the withdrawal rate was increased to 4%, some of the 50-year time horizons fell short, running out of money in as few as 33 years. However, as Bengen noted, most retirees have a time horizon up to about 25-30 years; as a result, he recommended a 4% initial withdrawal rate as a starting recommendation for clients. If clients wished to withdraw more, Bengen cautioned that the client may need to prune back the lifestyle in the future if a major event occurred.

Bengen also evaluated varying asset allocations, noting - to his surprise - that portfolio longevity was actually *worse* with higher equity exposures than with a 50/50 portfolio. When looking at the bad scenarios - how quickly did the money run out in the worst historical sequences - balanced portfolios actually did a better job of generating the returns necessary for the portfolio to survive inflation, without so much equity exposure that bear markets became catastrophic. However, increasing to 75% in equities gave up very little in bad scenarios, while significant improving portfolio longevity in many other scenarios. As a result, Bengen counseled that an optimal asset allocation would be as close to 75% in equities as possible, but no less than 50%, based on the client's "comfort zone" (i.e., risk tolerance) - in sharp contrast to the commonality, especially in the 1990s, to shift clients to significantly more conservative bond-centric portfolios as retirement approached.

In the years that followed, Bengen's results were replicated by other researchers who arrived at similar findings. Cooley, Hubbard, and Walz, of Trinity University, arrived at nearly

identical conclusions in their paper in 1998 (sometimes known as "the Trinity study" after the professors' university), although their work (which evaluated probabilities of success at various spending rates) also highlighted the fact that while 4% was generally a safe withdrawal rate, on average it also left a significant terminal value. Their research also supported the notion that equity exposure for retirement portfolios should be at least 50% in equities for longer retirement time horizons. The results were similarly replicated on a Monte Carlo basis in Pye (2000). Similarly, Ameriks, Veres, and Warshawsky (2001) found a 4.5% withdrawal rate was sustainable with a relatively low (less than 10%) rate of failure, as long as a moderately high equity exposure was used.

In the decade or so that followed, numerous additional researchers have all substantiated safe withdrawal rates in the neighborhood of 4% to 4.5%, which remains the 'baseline' in place today. Variations in the range are primarily attributable to the exact US historical dataset used, the timing of withdrawals, and whether "success" is determined at the 95%, 99%, or (in the case of historical analysis) 100% confidence levels.

However, one of the major criticisms of the safe withdrawal rate research is that it is generally predicated on the history of the US economy and markets. Whether done via historical simulations, or using Monte Carlo analysis, the results are still ultimately based on capital market assumptions derived from the economic and market experience of the US in the 20th century.

How much of an impact could a different future have? Blanchett and Blanchett (2008) showed that, because of the long-term nature of retirement time horizons, a 1% change in long-term returns can have a material impact on safe withdrawal rates, although a 1% change in the standard deviation of returns has far less of an impact. Accordingly, the driving force in a withdrawal rate of the future that might be different than the past would be a material change in US market or economic growth rates. Notably, though, periods of below-average returns are already accounted for in the existing safe withdrawal rate literature, and even emphasized in the valuation research (discussed later); what is required under the Blanchett research is not merely a moderate time period of reduced returns, but a fundamental shift in capital market return assumptions for an extremely extended (e.g., multi-decade) time period.

Pfau (2010) explored the impact of different potential return distributions by analyzing the safe withdrawal rates across 17 developed market countries for the past century. The results revealed that the historical equity returns for the US have in fact been "above average" relative to equity returns across all the developed countries, and that in fact a 4% safe withdrawal rate would have only worked in 4 of the 17 countries (including the US). Numerous countries with lower historical equity returns had safe withdrawal rates between 3% and 4%. And notably, a few countries experienced far more catastrophic results; the historical safe withdrawal rates for Germany and Japan were only 1.14% and 0.47%, respectively, associated with those who retired in the years shortly before those countries lost World Wars I and II.

On the other hand, it's notably that most countries in the sample had worst case scenarios below a 4% withdrawal rate tied directly to retirees during the first two World Wars, especially countries that were either directly ravaged by the war on their soil (e.g., France in World War II with a 1.25% safe withdrawal rate), or that lost a war (Italy, Germany, and Japan, with safe withdrawal rates of 0.47% to 1.56%). Outside of countries directly impacted by one of the two World Wars, the only country with a safe withdrawal rate below 3.6% was Spain, and in fact the average of the safe withdrawal rates amongst countries not directly damaged by World Wars was approximately 4%. Individual country variations appear to be driven by just how severe their worst stock market declines were, although notable not all countries' worst case scenarios are tied directly to early market crashes; sustained economic impairment or severe inflation sometimes play a role as well.

Overall, it appears that in the case of countries that do not experience significant wars on their soil, and experience merely "normal" economic cycles that do not fundamentally break their growth engine, a safe withdrawal rate in the range of 3.6% to 4.4% is consistent with the international historical data. However, in the event of a significant war, or a sustained breakdown in a country's economy beyond "normal" recessions or a moderate depression, all bets are off.

Safe Withdrawal Rate Baseline: 4% - 4.5% with a 'balanced' asset allocation (e.g., equity exposure of 40% to 70%). Consider a 3.6% base withdrawal rate with an expectation of sustained lower economic growth. Further adjustments merited only in light of expectations of a materially damaged economy due to war or other severe external shocks.

Asset Allocation Glide Paths

In the initial years that followed Bengen's original work, he received many questions regarding various assumptions from his research, especially with respect to asset allocation. The questions came from both other advisors, and clients with whom he was implementing his research. As a result, two years later Bengen published "Asset Allocation for a Lifetime" in the Journal of Financial Planning.

In this research, Bengen (1996) explored the impact of decreasing equity exposure over time, in an approach that later came to be called "glide paths" with the advent of target date funds. The basic principle was relatively straightforward - what happens if the client incrementally reduces equity exposure each year after retirement, ranging from a 0.5% reduction per year, up to a 3% reduction per year.

The results revealed that while lower equity exposures did reduce the risk of a severe portfolio downturn in later years, a downward glide to equity exposures actually *reduced* sustainable retirement income, as the steady replacement of a higher return asset with a lower return one ultimately eroded income sustainability over multi-decade time horizons. However, the adverse impact to retirement income was miniscule for reductions as high as 1.5%/year (whereas a reduction of 3% per year cut sustainable retirement income by a whopping 21%!), and Bengen actually concluded that a 1% per year reduction was a reasonable trade-off, dampening down volatility over time for a relatively negligible decline in sustainable retirement income. Thus, for clients who started retirement with an equity exposure of 63% (mid-way between Bengen's recommended 50% to 75% equity exposures), this led to a "rule of 128" where the client's optimal equity exposure would be 128 minus the client's age (leading to 63% equity exposure at age 65). More conservative clients might use a rule of 115 (leading to a 50% equity exposure at age 65). Notably, in later research that showed greater withdrawal volatility when measured on a quarterly (instead of annual) basis, Bengen (1997) cautioned against rules substantively higher than 130 (or more generally, equity exposures above 65%), due to the impact of the crash of 1929 for fourth quarter 1929 retirees.

Blanchett (2007) further extended Bengen's research on optimal glide paths, testing both conservative and aggressive starting points and with varying speeds of reducing equity exposure over time. Similar to Bengen, Blanchett found very limited benefit to glide paths that reduce equity exposure over time, acknowledging that in fact, "despite their simplicity, constant (static allocation) distribution glide paths proved to be remarkably efficient distribution strategies." More generally, Blanchett's research supported a 60/40 portfolio as a good anchor for a static allocation. Notably, though, because the dispersion of results increases over time, Blanchett also found that longer time periods are more sensitive to asset allocation and glide path assumptions than shorter time periods, and that when success is measured relative to variability, clients may prefer to be more conservative.

Asset Allocation Glide Paths: Consider very modest equity exposure reductions over time; limited apparent benefit in terms of allowing for a higher initial withdrawal rate.

Impact of Expenses

Pye (2001) was one of the first to examine the impact of investment expenses on safe withdrawal rates. Pye studied the impact of expenses by measuring how the safe withdrawal rate changed if the expected return for investments was reduced for an implicit expense ratio.

For instance, if the balanced portfolio's long-term return was 8%, then recalculating the safe withdrawal rate with an expected return of 7% and the same standard deviation would approximate the impact of 1% in investment expenses. Pye generally found that a 1% expense ratio translates into a safe withdrawal rate decline of 0.5%; thus, for instance, the 4% safe withdrawal rate based on index returns might be reduced to a 3.5% withdrawal rate after a 1% investment advisor fee. These results were supported by Kitces (2010), who similarly found that for balanced portfolios, each 100 basis point increase in investment expenses translated approximately 45% of that cost into a reduction in the safe withdrawal rate (e.g., a 1% expense ratio translated into a 0.45% reduction in the safe withdrawal rate; a 1.3% expense ratio would reduce the safe withdrawal rate by 1.3% x 45% = 0.59%).

Notably, the impact of investment expenses is not merely a direct reduction in the safe withdrawal rate - for instance, a 4% safe withdrawal rate minus a 1% fee is a 3% safe withdrawal rate. The reasons for this are two-fold. First of all, even a safe withdrawal rate of 4% is predicated on a *return* that is higher than 4%; because the withdrawal rate is assumed to adjust for inflation over time, and ultimately spend down principal, changes in returns do not necessarily translate 1:1 as reductions in the safe withdrawal rate. Secondarily, and perhaps more importantly, is the fact that the percentage-of-

assets-based fee itself recalculates and naturally adjusts downwards as a portfolio is spent. Thus, for example, while in year 1 a 4% withdrawal rate on a $1,000,000 portfolio is $40,000 and also has a $10,000 fee at 1%, in year 30 the inflation-adjusted withdrawal would be just under $100,000 and if the portfolio was being liquidated down to nothing and only had a remaining balance of about $100,000 for the last year's withdrawal, the investment fee would be only $1,000. Accordingly, while fees may have been equal to 25% of withdrawals in year 1, the fees are only 1% of the withdrawals in year 30 in a spend-down scenario (obviously, if the portfolio was going up, fees would be higher, but the portfolio would not be endangered, either). This naturally self-mitigating effect - where investment fees are higher in favorable environments but lower in spend-down scenarios - dampens the impact of investment expenses on safe withdrawal rates.

It is also important to note that Kitces (2010) showed an interaction effect between taxes and investment costs. Because investment costs reduce growth that will be subject to taxes, and/or taxes reduced the amount of growth that will be subject to future investment costs, the net result is that with even a moderate amount of taxation, the impact of investment costs translates not as a 45% relative reduction in safe withdrawal rates, but instead something closer to 35% (e.g., a 0.35% reduction in safe withdrawal rate per 1% of investment expenses).

Notably, this framework also offers a glimpse at how an investor who generates alpha might enhance safe withdrawal rates (although in practice many investors prefer to simply take alpha when earned, but not necessarily plan for it in advance). Just as a 1% expense ratio reduces the safe withdrawal rate by approximately 35 basis points, so too would 1% *excess* returns likely increase the safe withdrawal rate. Bengen (2006) framed this as the impact of the "superinvestor" who generates portfolio alpha.

Impact of Expenses: Reduce safe withdrawal rate by 35% of the associated expense ratio and investment advisory fees. Increase the safe withdrawal rate by 35% of any expected portfolio alpha.

Impact of Taxation

In his original work, Bengen was agnostic about taxes. However, his 1996 research began to explore the impact of taxation on the safe withdrawal rate from a taxable portfolio. Although the process is somewhat assumption intensive, Bengen concluded that at an average tax rate of 20%, the withdrawal rate was reduced about 0.3%, from 4.1% to 3.8%; an increase to a 35% tax rate further reduces the safe withdrawal rate down to 3.6%, a decline of approximately 0.5% from the tax-agnostic baseline. This may be a surprise to some, who would have expected that at a 20% tax rate, the safe withdrawal rate of 4.1% would be reduced by a full 20% (down to 3.3%, instead of only 3.8%). The difference is that in the end, not all growth and gains are taxable annually; furthermore, the spending of an after-tax account for retirement includes the liquidation of both taxable growth, and non-taxable principal; as a result, only a portion of the total amounts spent are actually taxable at all. In addition, the impact of taxation is mitigated by the fact that in general, tax liabilities are diminished in down years (as there are losses, not gains) and increased in up years (when the portfolio has generated gains); as a result, there is a natural self-dampening effect, as tax costs decline in declining markets (reducing the need for withdrawals while accounts are down) and increase in rising markets (when there is growth available to pay the tax liability).

Kitces (2010) found a slightly higher impact to taxation for safe withdrawal rates from taxable accounts. Using a more aggressive turnover assumption than Bengen (and also a higher dividend yield assumption that is taxed annually) but assuming no embedded gains at the start, Kitces found an average tax rate of 20% (on ordinary income and long-term capital gains/qualified dividends) reduced the safe withdrawal rate by as much as 0.66% for a moderate growth (60/40) portfolio, with a lesser tax impact for both more conservative and more aggressive portfolios. Clients facing higher tax rates in taxable accounts could have their safe withdrawal rate reduced by as much as 0.93%. Similarly, Pye (2001) found that the safe withdrawal rate was reduced by approximately 0.8% (from 4% down to 3.2%) with a 30% ordinary income tax rate, 20% capital gains tax rate, and a moderate level of turnover and dividend yield. Notably, all of these studies implicitly assume that the taxes are paid directly from the portfolio itself; thus, payments for taxes occur separately from and in addition to any withdrawals for spending itself.

On the other hand, for accounts that are pre-tax in nature - e.g., IRAs or 401(k) plans - the impact is both simpler, and more severe. For withdrawals that are entirely pre-tax, an effective tax rate of 20% really would outright cut the *spendable* portion of the safe withdrawal rate from 4.1% down to only 3.3%; a 35% tax rate would reduce it to 2.7%. These greatly diminished withdrawal rates are due to the fact that IRAs are not only taxed entirely as ordinary income, but

that the *entire* account is taxable; there is (typically) no material after-tax principle to liquidate as there is for an after-tax investment account.

Notably, Bengen also found that for retirements driven by taxable accounts, the optimal equity exposure was approximately 5% higher than otherwise. This appears not to be a consequence of the preferential tax rates for equities, but instead because the drag of taxation on returns necessitates a greater exposure to equities - as the higher returning asset - to make up for the tax drag. On the other hand, with higher turnover assumptions, Kitces (2010) actually found the opposite effect; because the drag of taxation was relatively higher on more equity-centric portfolios, the optimal asset allocation actually shifted to become more conservative (40% in equities instead of 60%) when accounting for the impact of taxation. This suggests that unless portfolio turnover is extremely low, the benefits of higher equities exposures above 40% may be limited, as the higher returns and associated withdrawal rates they allow are offset by the higher tax costs that result. In either case, though, for withdrawals from pre-tax retirement accounts, the optimal equity exposure remained the same; the client simply must adjust spending expectations to account the fact that the gross amount of the withdrawal must be netted down for taxation.

Impact of Taxes: Reduce safe withdrawal rate by approximately 0.25% for low-taxation clients, 0.5% for moderate-taxation clients, and 0.75% for high-taxation clients. Consider more conservative equity allocations for higher turnover portfolios. Apply effective tax rate on gross distribution for pre-tax accounts.

Varying Time Horizons

In his 1996 paper, Bengen also began to examine more thoroughly the impact of varying time horizons on sustainable retirement income. While the optimal withdrawal rate for a 30-year time horizon was approximately 4.1%, for a 20-year time horizon it rose to approximately 5.1%; on the other hand, for a 45-year time horizon, it dropped to only 3.5%.

Also notable was the fact that varying time horizons also altered the optimal asset allocation itself. While the same withdrawal rate remained at approximately 4.1% for asset allocations from 50% to 75% in equities, the optimal equity exposure for a 20-year retirement was only 50% in equities, showing a marked decrease in sustainable income for higher equity exposures. On the other hand, for the 45-year time horizon - with its longer period of inflation compounding expenses - the optimal equity exposure to sustain a 3.5% withdrawal rate was at the 65% to 95% level! However, later research by Bengen (1996) evaluating data on a quarterly basis - rather than annual - found that equity exposures above 65% in equities resulted in materially lower withdrawal rates, due to the impact of retiring in October of 1929, so some caution is merited unless clients want to assume there will never be a repeat of such a crash.

Similarly, Blanchett (2007) found a safe withdrawal rate of approximately 3.5% for a 40-year time horizon, and that the rate rose to nearly 5.5% for a 20-year time horizon. Blanchett's results also supported the notion that 40 year time horizons had greater safe withdrawal rates with nearly 60% in equities, while 30-year time horizons were optimal at equity exposures at the 30%-50% range; 20-year portfolios were optimal with only 10%-30% in equities. On the other hand, Pfau (2012) suggests slightly more conservative portfolios when evaluating based on a 95% probability of success, and in fact suggests equity exposures no higher than 30% over any time horizon (along with much lower withdrawal rates) for those who prefer a 99% confidence level.

Impact of Time Horizon: Increase safe withdrawal rate by 1% for 20-year time horizon (and make portfolio more conservative); decrease safe withdrawal rate by 0.5% for 40+ year time horizons.

Additional Diversification

The early research in safe withdrawal rates, from Bengen's early papers to the Trinity (1998) study, all assumed a two asset class portfolio of large cap stocks and intermediate government bonds, using the historical data available from the Ibbotson yearbook.

In 1997, Bengen published the 3rd article in his "series" in the Journal of Financial Planning, aptly entitled "Conserving Client Portfolios During Retirement, Part III". In this installation, Bengen explored the impact of adding more asset classes to the mix, including small cap stocks and Treasury bills.

Bengen's results showed that by adding 30% in small caps to an overall portfolio of approximately 60% in equities, the safe withdrawal rate was increase by approximately 0.2%, from just under 4.1% to slightly less than 4.3%. Although this may not seem like a large change, on a relative basis it represents a nearly 5% increase in lifetime annual spending.

Cooley, Hubbard, and Walz (2003) examined the effect of adding international diversification to the safe withdrawal rate. Somewhat surprisingly, they found only an extremely minimal benefit to including international diversification, with an improvement in probabilities of success of only a few percentage points (e.g., adding international exposure increased the probability of success for a 4% withdrawal rate from 95% to 96%). A similar result was found by Ervin, Filer, and Smolira (2005), who actually noted that extreme exposures to international stocks actually led to a decline in safe withdrawal rates. Nonetheless, Cooley et. al. suggest that some moderate amount of exposure to international stocks is merited, especially for longer time horizons, simply predicated on the existing finance research regarding the benefits of international diversification, even though their results suggest a general inability of global markets to substantively defend the US investor against US domestic bear markets. On the other hand, Kizer (2005) found a slightly greater benefit from international diversification (although it was still relatively moderate), and suggested the results validated inclusion of international diversification.

Cassaday (2006) evaluated the impact of a significantly more diversified portfolio, including not just large and small cap domestic stocks and an international component, but also REITs and commodities. Although his research was limited by the fact that he assumed a static 3% inflation rate, and only tested returns since 1972 (when notably, the historical "worst case" scenarios were tied to starting dates around the Great Depression or the mid-to-late 1960s), he nonetheless found sustainable withdrawal rates as high as 7%. Although testing through tougher time periods (e.g., a deflationary environment) and adjusting spending for actual inflation would have almost certainly moderated this result, the Cassaday research still supports the notion that greater diversification may be supportive of higher withdrawal rates, especially the inclusion of commodities and real estate in the midst of inflationary environments.

Regarding the fixed income portion of the portfolio, Kizer (2005) suggested that international bond exposure may assist in overall sustainable retirement income, although he did not quantify the effect separate from the benefits of international equity diversification. Bengen (1997) found that adding Treasury bills to the fixed income portfolio generally reduces success due to lower returns, especially for more conservative portfolios, although the impact of 10% in such cash equivalents is virtually negligible and may help to facilitate spending distributions.

Diversification Benefits: Increase safe withdrawal rate by 0.5% to 1.0% for significant multi-asset class diversification. Notably, though, the exact value of diversification is still largely theoretical, as there is limited history to effectively test the benefits of particular asset classes, and some diversification benefits may be duplicative and/or diminishing (e.g., adding the 5th equity-oriented asset class may have limited additional diversification value after the first 4).

Ongoing Spending Changes & Decisions Rules

In a 4th paper in his series published in 2001, Bengen explored the implications for safe withdrawal rates assuming a retiree adjusts spending downwards in later retirement years, rather than simply maintaining the exact same inflation-adjusted spending for life.

For instance, what happens if retirees adjust their spending for inflation up to age 75, inflation minus 4% (e.g., if inflation was 3%, the retiree's expenses are adjusted by -1%) from age 75 to 85 (as their lifestyle and activity slows down), and then inflation minus 2% from age 85 onwards (as some discretionary expenses have been fully eliminated at that point). By the end of the path, the client with decreased spending in later years is spending about 45% less than the traditional inflation-adjustments-for-life client. And as a result, the client's safe withdrawal rate climbs from 4.1% to nearly 4.8%, an increase in the starting spending amount of almost 15%. On the other hand, it's notable that the starting spending increases "only" by 15%, even though final

spending in nominal dollars is almost 45% lower by the end; this disparity occurs because the retirement longevity overall is still disproportionately impacted by withdrawals in the first half of retirement, where spending is still increasing in the same manner.

Bengen's 2001 research also explored the implications of a more dynamic strategy to adjust withdrawals based on actual portfolio performance. Noting that simply recalculating withdrawals at the start of each year would produce untenable spending volatility, Bengen suggested a method where withdrawals increase during bull markets, but not more than 25% above the real value of the first year's withdrawal, and can decline during a bear market, but not more than 10% below the real value of the initial withdrawal. Following such a dynamic spending methodology, Bengen found that the portfolio could sustain a withdrawal rate of nearly 4.6%, more than 10% higher than the baseline 4.1% safe withdrawal rate.

An extension of Bengen's ongoing adjustments approach was explored by Guyton in 2004 (and a follow-up paper by Guyton and Klinger in 2006). Using both a historical analysis of a retiree on the cusp of the 1973-74 bear market (in the 2004 paper) and then a Monte Carlo analysis with a wider range of historical data (the 2006 paper), the analysis evaluated how a series of "decision rules" could improve safe withdrawal rates. The decision rules, determined up front, would dictate how client spending would change from year to year. The baseline was still a portfolio where withdrawals would increase annually for inflation; however, the decision rules capped inflation increases at 6%, and eliminated any inflation increase if the prior year's total return on the portfolio was negative (roughly analogous to a worker not receiving a CPI adjustment to salary when the company isn't profitable for the year). More significantly, Guyton also included what he called a "Capital Preservation Rule" and a "Prosperity Rule" that would increase or decrease current spending by 10% in nominal dollars if the current withdrawal rate (based on spending and the portfolio at that time) deviated by more than 20% from the original spending path in the first 15 years. Thus, for example, if the initial withdrawal rate was 4%, then the adjustment thresholds would be 3.2% and 4.8% (+/- 0.8%, which is 20% of 4%); accordingly, if after 5 years the client's spending was up to $50,000/year with inflation but the portfolio had dropped down to $950,000 - a current withdrawal rate of 5.26% - the client's spending would hit the upper "guardrail" and be cut by 10% from $50,000 down to $45,000 to get it back on track again.

Following the decision rules approach, Guyton found that over a 30-year time horizon, the portfolio could sustain a safe withdrawal rate of 5.2% using all of the rules and a simple 65/25/10 allocation of equities/fixed/cash at a 99% probability of success; the associated spending pattern ultimately added up to 102% of the client's purchasing power compared to the baseline scenario (due to the prosperity rule eventually ratcheting up spending over time, even if the spending was initially cut due to a bear market scenario). Notably, a 5.2% withdrawal rate was also sustainable over a 40-year time horizon as well. In follow-up research by Klinger (2007), it was observed that applying the capital preservation rule with larger potential cuts actually allowed for higher withdrawal rates - ostensibly because the portfolio was more responsive in making significant adjustments in the face of significant downward market volatility.

Overall, Guyton and Klinger's works found that clients who were willing to cut spending in difficult markets, and constrain spending (by forgoing inflation adjustments) in flat markets, were ultimately able to achieve comparable lifetime spending, but at a starting level that was more than 20% higher. In fact, it is notable that it would take a combination of several cuts and/or freezes just to drop the higher initial spending levels down to the original 4.1% withdrawal rate, although on average Guyton's rules did in fact apply 2 cuts and 6 spending freezes (ultimately made up by an average of 7 prosperity rule increases).

A later extension of this framework was done by Frank, Mitchell, and Blanchett (2011), who suggested that a more effective approach for "guardrails" might be to set a probability-of-failure threshold; for instance, if the current plan entails a probability of failure in excess of 30%, spending must be adjusted downwards. The advantage of this approach is that a probability of failure automatically adapts to the client's remaining time horizon and current withdrawal rate - an important distinction, as the withdrawal rate would naturally be expected to rise simply due to the shortening time horizon as the client ages. This approach allows for a distinction between a rising withdrawal rate due to age, versus a rising withdrawal rate due to rising risk of asset depletion that requires adjustment.

Ongoing Spending Changes: Increase safe withdrawal rate by 0.5% for clients who can make modest spending changes in bear markets and/or who plan to decrease spending in later years; increase safe withdrawal rate by 1.0% for clients who can make more substantive (e.g., 10%+) spending cuts in bear markets and/or who plan more significant cuts in later years.

Impact of Risk Tolerance

In Bengen's original research, the goal was to find a withdrawal rate that was so low, it survived any disaster scenario found in US economic history; in other words, the withdrawal rate that had a 100% probability of success (i.e., one that survived 100% of available historical scenarios). Subsequent studies that reproduced Bengen's work on a Monte Carlo basis affirmed that the probability of success likely isn't 100% - after all, it's always at least *possible* to have a market or economic disaster in the future that was worse than anything found in history - but that a starting withdrawal rate of 4% to 4.5% was still 'extremely safe' as a starting point.

However, as noted in Kitces (2012), the reality is that a probability of failure typically doesn't represent the likelihood of a total failure of the plan, but merely the likelihood that the client will have to make some adjustments along the way to get back on track or extend the time horizon for a spenddown. The greater the likelihood of "failure," the more likely it is that an adjustment may need to happen, and the more severe the potential adjustment could be.

Accordingly, it stands to reason that clients who are more willing to make adjustments if necessary, and/or are more tolerant of the risk that adjustments could be required, might consider choosing a higher withdrawal rate than the one that is safest with a minimal/negligible probability of "failure." To some extent, this was already shown in the research by Guyton (2004) and others about the impact that planned spending adjustments can have on the initial withdrawal rate. The difference, though, is that Guyton focused on the rules necessary to determine the adjustments, not the client's tolerance for the risk of adjustments themselves.

A more recent approach to attempt to evaluate the trade-offs of higher withdrawal rates in exchange for accepting higher probabilities of failure was done by Spitzer, Strieter, and Singh (2007), who showed that investors willing to accept a 25% probability of failure/adjustment could consider withdrawal rates as high as 5.5% - 6%, supported by a greater allocation to equities. Similarly, Cooley, Hubbard, and Walz (2007) suggested safe withdrawal rates above 5% for those with fairly aggressive equity exposure (e.g., 75% of the portfolio) willing to accept a 25% probability of failure.

A further extension of this approach is to try to quantify how clients make financial decisions based on their tolerances and willingness for trade-offs using economic utility functions. For instance, Tomlinson (2012) showed how a utility function and the client's loss aversion might alter a retirement income strategy between equity, bonds, and a lifetime annuity. Finke, Pfau, and Williams (2012) further extended a utility framework to show that clients who have higher risk tolerance and are therefore assumed to be more flexible regarding spending might deliberately choose materially higher withdrawal rates, leading to a safe withdrawal rate as high as 7%, along with greater equity exposure.

The fundamental point is that clients who have greater tolerance for risk potentially do not need to identify planning strategies that reduce the risk of failure to near-zero negligible levels, especially as more risk-tolerant clients may be more willing to make spending and lifestyle changes to accommodate an unfavorable sequence, should one occur.

Impact of Risk Tolerance: Increase safe withdrawal rate increase of 0.5% to 1.0% for clients with significant tolerance for risk and willingness to make spending changes. Consider higher adjustments for very risk-tolerant clients with significant spending flexibility.

Impact of Market Valuation

One significant challenge of the existing safe withdrawal rate research is that the application of the 4% rule is surprisingly sensitive to client's starting account balance on the date of retirement. This was dubbed by Kitces (2008) as "the timing paradox" and was expressed as follows:

> *Imagine two clients, the Retirenows and the Notquiteyets, who both have a $1,000,000 portfolio available for retirement. The Retirenows begin retirement immediately, and use an initial withdrawal rate of 4.5% to produce $45,000/year of real spending (increasing each future year for inflation). The Notquiteyets, on the other hand, decide that they wish to work for one more year, and plan to retire next year instead.*
>
> *Over the ensuing year, a bear market emerges, and at the end of the year both portfolios have experienced 15% market declines. The Notquiteyets, now ready to retire, can safely spend $38,250 (which is 4.5% of their now-$850,000 portfolio) adjusted each year for inflation. In the meantime, with a 3% inflation adjustment on their original safe withdrawal amount, the Retirenows are informed that they can safely spend $46,350.*

A strange paradox had emerged. At the beginning of year 1, the Retirenows and the Notquiteyets both had $1,000,000. Both experienced the same investment returns. Yet by applying the same methodology, the Notquiteyets were informed that they can safely spend only $38,250/year for life (adjusted for inflation), while the Retirenows can safely spend $46,350/year for life (inflation-adjusted from that year forward)! The Retirenows are able to safely spend almost 21% more than the Notquiteyets, despite the identical investment results. And in fact, the disparity is even more shocking; because the Retirenows also spent money in the first year, the reality is that not only is their safe spending in year 2 a whopping 21% higher than the Notquiteyets, but their portfolio value is actually lower. After all, the Retirenows didn't just experience the market decline; they also took a first year spending withdrawal!

The Kitces solution to this timing paradox was that clients who retire after a bear market should be eligible for a higher withdrawal rate (albeit on a lower account balance) than those who retire before a market decline, because of the change in market valuation.

Using a base safe withdrawal rate of 4.5%, Kitces found that such withdrawal rates are actually only necessary when market valuation - as measured by Shiller's P/E10 ratio - is in the upper quintile of its historical range, which corresponds to a P/E10 greater than 20. If the P/E10 is less than 20, then the safe withdrawal rate becomes at least 5.0%. If P/E10 ratios decline to a level below 12 - associated with the bottom quintile of historical valuation levels - the safe withdrawal rate rises to 5.5%, with a significant likelihood of future spending increases.

Kitces' withdrawal rate paradox was further explored by Pfau (2011a), who expanded the framework (and predictive value) to analyze dividend yields and nominal Treasury bond yields, in addition to P/E10 ratios (or actually the reciprocal E10/P as a measure of earnings yield, which Pfau called EY10). Pfau's model generally affirmed the fluctuations in the safe withdrawal rate, which correctly anticipated the low withdrawal rates that followed the middle of the first decade of the 1900s, the late 1920s, and the mid 1960s. The Pfau model also suggested that indeed, many environments are capable of supporting higher withdrawal rates than the current 4% rule. On the other hand, Pfau's research also noted that the current environment may require withdrawal rates as low as 2%, due to the unique combination of low interest rates, low dividend yields, *and* high valuation levels present over much of the past decade (due in no small part to how excessive P/E10 ratios became in the late 1990s, even relative to historical bubbles). In a follow-up article, Pfau (2011b) also noted that because favorable (unfavorable) retirement time periods are often the result of unfavorable (favorable) pre-retirement periods that precede them, the overall stability of required retirement saving and accumulation may actually be less uncertain than commonly believed.

A further analysis by Kitces (2009), explored the impact of changing not only safe withdrawal rate recommendations based on market valuation, but also changing the asset allocation itself. For instance, Kitces tested a scenario where the retiree's starting portfolio was 60/40, but the allocation was increased to 80/20 in favorable valuation environments, and reduced to 40/60 at unfavorable valuation levels. The results revealed an increase of safe withdrawal rates by approximately 0.2%, across all starting valuation ranges, by engaging in the more dynamic allocation strategies, even if returns themselves were not increased. A subsequent paper by Solow, Kitces, and Locatelli (2011) showed that a more strictly implemented tactical asset allocation strategy (i.e., only trading selectively at market valuation extremes) can yield both absolute and risk-adjusted return enhancements as well.

Impact of Market Valuation: Increase safe withdrawal rate by 0.5% in moderate/average valuation environments, and 1.0% in favorable valuation environments. Add 0.2% to safe withdrawal rate for portfolios that will tactically reduce exposure in high valuation environments and increase equities at favorable valuations. Consider reducing the safe withdrawal rate in extreme combinations of high valuation and low interest rate environments.

Use of Annuities

Another line of research in safe withdrawal rates, started by Ameriks, Veres, and Warshawsky in 2001, was to explore the impact of partially annuitizing a portion of the client's wealth in a single premium immediate annuity at the start of retirement.

Ameriks et. al. tested scenarios where the client, assumed to be a single male age 65, annuitized either 0%, 25%, or 50% of the portfolio, at then-current annuitization rates. The results indicated a moderate but material reduction in the probability of failure for a 4.5% withdrawal rate by supplementing with an annuity, with a greater benefit for annuitizing more of the portfolio.

Unfortunately, though, when Ameriks et. al. performed their study, it was suggested that the annuitization rates used would likely be reasonable and sustainable. However, in reality over the decade that has followed, immediate annuitization rates have dropped significantly. The research assumes that the annual payment stream a 65-year-old male could generate would be equal to approximately 9% of the starting portfolio (not adjusting for inflation); in today's marketplace, though, the amount is barely 7%, which may create a material diminishment to the effectiveness of the annuity as the client would be required to rely much earlier on material withdrawals from the portfolio. On the other hand, Pfau (2011a) has suggested that at today's low interest rates, especially when combined with high valuations, even the 4% safe withdrawal rate may not be a safe baseline.

Regarding annuities, it is unclear whether a portion of the Ameriks et. al. effect may simply be due to the systematic liquidation of a "fixed" portion of the portfolio first and an associated upwards glide path in implied equity exposure.

Research by Pfau (2011c) has also explored whether current variable annuity living benefit riders may enhance safe withdrawal rates. Pfau generally found that due to the expenses of such guarantees, and the lack of inflation adjustments to the benefits, the GLWB guarantee may prevent clients from running out of money entirely, but does not necessarily increase the sustainable withdrawal rate in the first place.

Use of Annuities: Uncertain benefit to partial annuitization in current rate environment; annuitization clearly prevents total loss of income, but may not necessarily lead to a higher initial withdrawal rate. However, there is little evidence to suggest a lower withdrawal rate, either, as long as the annuity contract is reasonably priced.

Leaving a Legacy

The traditional safe withdrawal rate research is predicated on the idea that in the worst case scenario, the assets will last for exactly the targeted time horizon and no more, with the final dollars extinguished in the final year. However, in practice some clients may have legacy goals, and wish to leave assets to the next generation.

Bengen (2006) showed that a goal of leaving 100% of the desired principal at the end of a 30-year time horizon reduces the safe withdrawal rate by just under 0.2%. Although this number may seem surprisingly low to some, bear in mind that relative to a safe withdrawal rate of approximately 4%, this represents a spending cut of nearly 5% of the starting amount, per year, for life (where 0.2% / 4% = 5%).

In some cases, planners may wish to make a 'legacy' adjustment not merely to accomplish a legacy goal, but simply to increase the likelihood of having an amount of wealth remaining at the end of the time horizon, in case the client lives longer than expected. On the other hand, it's important to note that the safe withdrawal rate merely stipulates no expected legacy in the *worst case* scenario; even Bengen's original research showed that while there is no wealth left remaining in the most adverse scenarios, in a whopping 96% of the cases the client's *entire* starting principal remains!

Leaving a Legacy (Or Hedging Against Longevity): Reduce safe withdrawal rate by 0.2% (further adjusted up or down for larger or smaller legacies)

Bringing It All Together

So how does a planner integrate all of these various segments of the safe withdrawal rate on behalf of a client?

Bengen (2006) suggested viewing the issue as a "layer cake" that starts with a base withdrawal rate that is subsequently adjusted up or down, based on the particular layers that are relevant to the client's situation.

For instance, assume a conservative client starts with a safe withdrawal rate of 4.0%. The client pays total fees of 1.2%, reducing the safe withdrawal rate to approximately 3.6%. The client also faces a moderate tax rate of 15% on capital gains and 25% on ordinary income, which reduces the safe withdrawal rate by another 0.5%, down to 3.1%. (*Editor's Note: As discussed earlier, this ultimately assumes the investment accounts are taxable accounts. If they are pre-tax retirement accounts, the 25% ordinary income tax rate would be applied directly to the gross withdrawal to determine a net spendable amount.*)

However, the client couple is already in their late 60s, and decides that a 25-year time horizon is sufficient, increasing the safe withdrawal rate by 0.5%. In addition, their portfolio is extremely well diversified across

multiple asset classes, further pushing their safe withdrawal rate 0.75% higher, to a total of 4.35%.

The couple also has moderate flexibility for making spending cuts, as they are willing to cut travel and some other expenses for a few years if markets are difficult. This spending flexibility increases their safe withdrawal rate by another 0.5%, up to 4.85%. However, given their conservative nature, the couple still prefers an extremely high probability of success and to minimize the risk of spending cuts, so they do not want to increase their safe withdrawal rate further based on their risk tolerance.

In addition, the environment the clients are retiring in would be one characterized by fairly average long-term valuation levels, which increases their safe withdrawal rate by 0.5%. Furthermore, they are more concerned about preserving their assets than being exposed to maximal growth, and are willing to adjust their asset allocation tactically to reduce equity exposure if valuations rise excessively, increasing their safe withdrawal rate by an additional 0.2%. This brings their total safe withdrawal rate up to 5.55%.

Given their conservative nature, though, the clients ultimately decide that they would like to hedge against longevity by reducing their spending to increase the likelihood of a legacy to be available for long life (or "at worst" to leave to their children, reducing their safe withdrawal rate by 0.25% to 5.3%.

Thus, as a result of the various safe withdrawal rate adjustments available with the framework, the client's safe withdrawal rate would be 5.3% of their current assets, with that dollar amount adjusted annually for inflation in future years, as shown below.

Base Withdrawal Rate	**4.0%**
Negative Adjustments	
1.2% Fees	-0.4%
Moderate Taxes	-0.5%
Legacy/Longevity Hedge	-0.25%
Positive Adjustments	
25-year Time Horizon	0.5%
Significant Diversification	0.75%
Some Spending Flexibility	0.5%
Moderate Valuation Environment	0.5%
Tactical Asset Allocation	0.2%
Final Withdrawal Rate	**5.3%**

More generally, the process of applying safe withdrawal rate adjustments and "baking the layer cake" as Bengen put it, might be applied as follows:

Base Withdrawal Rate	**4.0% - 4.5%**
Adjustments	
Fees/Alpha	-1% to 1%
Taxes	-0.25% to -0.75%
Legacy/Longevity Hedge	0% to -0.4%
Time Horizon	-0.5% to 1%
Diversification	0.5% to 1%
Spending Flexibility	0% to 1%
Risk Tolerance	0% to 1%
Valuation Environment	0% to 1%
Tactical Asset Allocation	0% to 0.2%
Final Withdrawal Rate	**Sum Total of Adjustments**

Caveats

Although the "layer cake" approach of safe withdrawal rates does allow for planners to adapt a safe withdrawal rate to a client's specific circumstances, there are several important caveats to be aware of.

The first and most significant is that many of the factors discussed here were evaluated in separate research studies, and it is not necessarily clear whether they are precisely additive. For instance, as noted in the section on diversification, while small caps, international stocks, and other asset classes may all provide some diversification benefit, there may be some duplicative benefits as well (e.g., adding international stocks may be less of a diversification benefit if small cap stocks were already added). In theory, this may be true for other factors as well; for instance, the impact of taxes may change with certain tactical asset allocation strategies that reduce volatility; the consequences of an altered time horizon may be a moot point if the portfolio generates alpha, or is exacerbated by fees and taxes. In addition, some factors may simply have less impact because of others; for instance, the diversification effect may be less relevant for 20-year time horizons that have less exposure to non-fixed investments in the first place, and tactical asset allocation shifts may be less relevant as well (on the other hand, both factors may be more relevant at 40-year time horizons!). These 'interaction effects' have still only received limited study thus far, but generally suggest some caution about clients trying to stack too many additive effects on top of each other.

In addition, it is also still possible that some factors have not yet been tested or cannot be clearly applied, such as the impact of more dramatic planned spending adjustments, or the consequences of extreme valuation environments or economic catastrophes not modeled anywhere in the historical data, or a clear conclusion on the impact of annuities.

And from a more generalized level, safe withdrawal rates are ultimately still built on the assumption of a level spending amount that adjusts annually for inflation (except where explicitly flexible via spending adjustments). It still does not apply clearly in the case of significantly variable expenses or assets, such as a client retiring at age 55 who must take significant withdrawals for 10 years until Social Security and a pension begin, after which withdrawals are much more modest, and the portfolio is augmented with the sale of a second home in 20 years. Such scenarios must ultimately still be evaluated with other tools that can target more detailed and unique client scenarios, such as Monte Carlo analysis.

Conclusion

The safe withdrawal rate has come far over the past 20 years. What started as an initial framework to determine a spending level that would be "safe" because it survived all historical scenarios for a 30-year time horizon has expanded as subsequent researchers build upon the original research.

In practice, this makes the safe withdrawal rate somewhat more complex than Bengen's original formulation. On the plus side, though, the very process of applying adjustment factors based on the client's individual facts and circumstances allows for a generalized safe withdrawal rate framework to be applied to specific client scenarios, allowing any client to know not just "what is the safe withdrawal rate" but "what is the safe withdrawal rate for their goals in particular."

Ultimately, many planners will likely still supplement a safe withdrawal rate evaluation with an ever-more-client-specific Monte Carlo analysis; however, once safe withdrawal rate factors are applied properly, the results may already be accurate enough to make further projections a moot point!

What did you think?

Hopefully you found this latest issue of The Kitces Report to be of value to you. However, since it is produced for you, the reader, we would like to hear from you about how the style, format, and content of the newsletter could be further improved to make it more valuable for you.

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what you think by emailing us at
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Thanks in advance
for sharing your thoughts!

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